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INVERSE CASCADE IN FRACTAL TURBULENCE (VORTEX-FRACTONS)

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A direct connection between the properties of the inverse cascade of energy and the fractal properties of turbulence is established.

Introduction. The transport of energy from small-scale to large-scale motions attracted attention long ago. A large body of literature is devoted to this topic (see, for example, [1-6]), and it can be asserted that its existence has been reliably established by experiment. It is considered that the inverse cascade is a characteristic feature of large-scale processes in quasi-two-dimensional turbulence. However, the physical nature of the inverse energy cascade is still not well understood, and there is a great deal of experimental material, obviously connected with the inverse cascade, which needs interpretation. Evidently, one of the fundamental points that is unclear here is the connection between the inverse cascade and the fractal character of turbulence. The difficulty in the theoretical analysis of this problem is due to the specific fractal structure in two-dimensional turbulence (for example, see [5, 7]). Below it will be shown that with the suppression of the fractal character of the motion, the inverse energy cascade in two-dimensional turbulence is also suppressed at large scales. That is to say, the very existence of the cascade turns out to be caused by the fractal character of the turbulence.

The carriers of the inverse cascade are the large-scale, localized fractal formations, closely linked to the fracton dimension of the fractal processes [8]. Evidently, the characteristic quasi-horizontal vortices with structures on the scale of ~1-100 km, which have been observed in the ocean [9], are such vortex-fractons (see below). In magnetohydrodynamic turbulence, it is still not possible to visualize vortex-fractons. However, the spectral and integrated characteristics of the processes, for which these vortices are responsible, have been measured in numerous experiments. Below, a comparison with these data will be made. This comparison indicates that for both the oceanic turbulence and the MHD turbulence, the inverse energy transfer to large scales is linked to processes of a fracton nature. A connection is established between the fracton dimension of the turbulence D_f [8] in the two-dimensional case and the low-frequency scaling spectrum of the kinetic energy of pulsation

$$E(\omega) \sim \omega^{D_f - 3}. \tag{1}$$

For the universal (approximate) value $D_f = 4/3$ [8] (Alexander–Orbach), relation (1) gives

$$E(\omega) \sim \omega^{-5/3}, \tag{2}$$

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that is, the well-known spectral law of the Kolmogorov type (see, for example, [1-6]). With suppression of the fractal character, $D_f \to 2$ (in the two-dimensional case), and from (1) we have

$$E(\omega) \sim \omega^{-1}. \tag{3}$$

In this case, the inverse transfer of energy is also suppressed.

The action of both laws (2) and (3) is observed in numerical and natural experiments in the appropriate conditions.

One can also examine the damping of the total pulsating energy field of the velocity in time, $u^2(t)$ in those cases, when it is determined primarily by the low-frequency range. For $D_f = 4/3$, we obtain

$$\overline{u^2} \sim t^{-2/3}, \tag{4}$$

and for the case of suppressed inverse cascade ($D_f \to 2$)

$$\overline{u^2} \sim t^{-1}. \tag{5}$$

Laws (4) and (5) are also observed in the MHD experiments of various authors.

If indeed we still make use of the Kolmogorov hypothesis [1, 2]

$$E(\omega) \sim \left(\frac{d\overline{u^2}}{dt}\right)^{2/3} \omega^{-5/3}, \tag{6}$$

then for the universal ($D_f = 4/3$) case, we obtain from (4):

$$E(\omega) \sim t^{-10/9} \omega^{-5/3}. \tag{7}$$

A law of such form is satisfied (both in t and in ω) in well-known MHD experiments [10] for low-frequency (large-scale) pulsations of the velocity field.

On the whole, it can be said that vortex-fractons evidently determine the local, as well as the integral dynamics of quasi-two-dimensional, large-scale turbulent processes causing the inverse energy cascade, and its suppression ensures an equilibrium situation.

1. The probability density of finding a particle located in a turbulent medium, $p(\mathbf{r}, t)$ is described by the equation

$$\partial p/\partial t = \operatorname{div} \chi \nabla p - \mathbf{u} \nabla p, \tag{8}$$

where χ is the diffusion coefficient, and $\mathbf{u}(\mathbf{r}, t)$ is the pulsating velocity field. The first term on the right-hand side of (8) describes diffusion, and the second, convection in the velocity field. If the velocity field is stochastic (turbulent motion), then the solution to (8) will also be stochastic. The vorticity of the velocity field $\Omega = \operatorname{rot} \mathbf{u}$ is highly nonuniform in turbulent motion. Its properties are readily described with the help of fractal methods [6, 8, 9, 11]. The geometric structure of the vorticity field is characterized by a series of universal parameters. These parameters are of the type: the fractal dimension D, the fracton (spectral) dimension $D_f = 2D/D_w$, where D_w is the dimension of the internal wander within the fractal [8]. According to traditional interpretation, the region occupied by a fluid with significant vorticity, is precisely the region of turbulence. The remaining parts of the fluid are considered to be laminar, so that the fractal characteristics of the vorticity field are, in essence, the fractal characteristics of the turbulence.

A particle located in the fluid and described by Eq. (8), wanders through the fractal vorticity field without leaving it [12]. The fractal dimension of the internal wander is D_w. The low-frequency characteristics of these random walks have a number of universal properties. The dimension D_r of the set of moments in time, at which the particle returns to its initial point is [8]:

$$D_r = 1 - D_f/2. \tag{9}$$

As a result, the time differential in the fractal regime is (for sufficiently large t)

$$(dt)_f \sim t^{-D_r} dt. \tag{10}$$

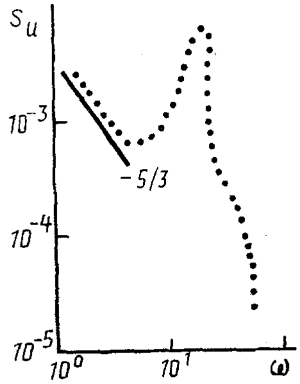

Fig. 1

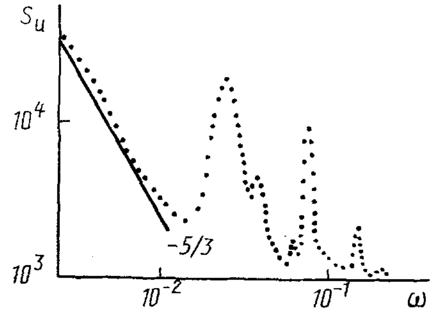

Fig. 2

Fig. 1. Characteristic behavior of the low-frequency spectrum in turbulent MHD flows.

Fig. 2. Spectral density of the kinetic energy of the horizontal velocity components of intermediate-scale oceanic flows.

If we switch to the dual of t, the frequency variable ω, then

$$(d\omega)_f \sim \omega^{-D_r} d\omega \tag{11}$$

(now for sufficiently small ω), that is, the probability density of encountering the frequency range [ω, ω + dω] in this process is

$$p(\omega) \sim \omega^{-D_r}. \tag{12}$$

From this [and from (9)], the spectral density of the random process p(t) for small ω is

$$S_p(\omega) \sim \omega^{D_f - 1}. \tag{13}$$

In two-dimensional turbulence, there is only one component of the vector vorticity $\mathbf{\Omega} = \operatorname{rot} \mathbf{u}$, which is normal to the plane of the motion. It is well known that this component satisfies (8), as does p(r, t) [1-3]. Thus, it can be assumed that for sufficiently large t (small ω), the spectral characteristics of p(r, t) and Ω(r, t) will coincide. That is

$$S_\Omega(\omega) \sim \omega^{D_f - 1}. \tag{14}$$

On the other hand, it is known that the spectral density of the pulsating energy rate $S_u(\omega)$ is related to $S_\Omega(\omega)$:

$$S_u(\omega) \sim \omega^{-2} S_\Omega(\omega). \tag{15}$$

Then from (14) and (15) we obtain

$$S_u(\omega) \sim \omega^{D_f - 3}. \tag{16}$$

2. The spectral (fracton) dimension D_f, generally speaking, depends on the fractal type and the topological dimension of the space in which the fractal is embedded. However, in the majority of cases, it does not differ greatly from 4/3 (the Alexander–Orbach hypothesis [8]). Upon substituting such a universal value for D_f in (16), we obtain a spectrum for the velocity field of the Kolmogorov form $S_u \sim \omega^{-5/3}$. Experiments on two-dimensional turbulence (numerical and natural [2]) obtained low-frequency portions of the spectrum with such a dependence. Natural MHD experiments and observations of oceanic turbulence are of the greatest interest for confirmation of the proposed connection. Among the MHD experiments, [10, 13] can be singled out, in which turbulence was generated in mercury with the help of hydrodynamic grids. In this case, a strong uniform magnetic field was imposed on the motion (the turbulence was generated in the field). In the low-frequency region (to the left of the generating frequency), a spectrum close to (2) was observed. For illustration, Fig. 1 shows such a spectrum, taken from [13]. The damping law for the pulsation energy in time was also observed in these experiments (on these laws, see below).

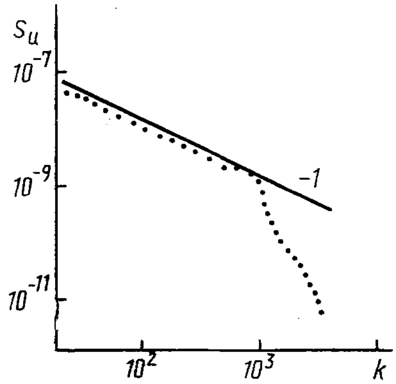

Fig. 3

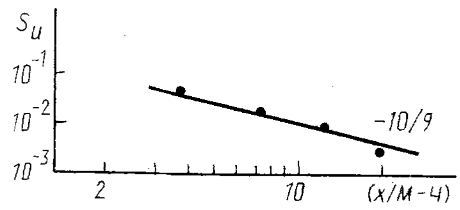

Fig. 4

Fig. 3. Energy spectrum of quasi-two-dimensional MHD flows.

Fig. 4. The damping of pulsation energy with time.

Since the probability density of finding the frequency in the interval [ω, $\omega + d\omega$], in the range of ω is considered here, and is given by (12), then for $D_f = 4/3$, it has the form [see also (9)]

$$p(\omega) \sim \omega^{-1/3}. \tag{17}$$

Thus, in this range, $p(\omega)$ grows with decreasing ω. As a consequence, the pulsation energy here will be transferred from high frequencies to low, to the side of the most probable frequencies (as in all systems with different probabilities of realizing different states).

Figure 2, taken from [9], shows the spectral density of the kinetic energy of horizontal velocity components, measured in the ocean. A straight line of slope $-5/3$ has been drawn to show the behavior of the spectrum in the low-frequency region. In these observations, it was also noted that the energy of macropulsations goes into increasing the energy of the mean motion, that is to say, there is an inverse energy cascade. Characteristic turbulent vortices (fractons) of dimension ~1-100 km were also noted in the study. With suppression of the fractal character in two-dimensional turbulence, $D \to 2$ and $D_w \to 2$. Consequently, in this case, $D_f = 2D/D_w \to 2$. From (9) and (12) we then conclude that when the fractal character is suppressed, $D_r \to 0$ and $p(\omega) \to$ const.

Thus, when the fractal character is suppressed, the transfer of energy to the low-frequency range, due to the different probabilities of frequency realization, is decreased. From this, it can be concluded that it is precisely this fractal character of the turbulence which causes the inverse transfer of energy in the low-frequency range for two-dimensional turbulence. The energy spectrum also changes when the fractal character is suppressed. Since $D_f \to 2$, then from (16) we obtain $S_u \sim \omega^{-1}$. Such equilibrium spectra were obtained in numerical experiments on two-dimensional turbulence (see, for example, review [2]). We note here the experiment described in [14], in which developed turbulence was placed in a strong magnetic field. (This distinguishes the experiment from the previously mentioned experiments, in which the turbulence was generated in the magnetic field.) During such containment, the three-dimensional fractal structure, characterized by steep velocity-field gradients, is suppressed by the strong interaction with the external magnetic field (and not built up under the field, as in the case of generation of turbulence in the magnetic field). Figure 3, taken from [14], shows experimental data obtained in such circumstances for the energy spectrum ($k \sim \omega$). The straight line is drawn in to mark the adherence to law (3).

3. We now examine damping in time of the integral characteristics of turbulence.

The diffusion in a turbulent fractal of a smooth envelope of the function $p(\mathbf{r}, t)$, which we denote by $P(\mathbf{r}, t)$, is described by the equation [8]

$$\partial P/\partial t = \frac{1}{r^{D-1}} \frac{\partial}{\partial r} \chi r^{D-1-\theta} \frac{\partial P}{\partial r}, \tag{18}$$

where $\Theta = D_w - 2$. For large t, the asymptotic solution to this equation has the form [8]

$$P(r, t) \sim t^{-D/D_w}. \tag{19}$$

By using the analogy between p(**r**, t) and Ω(**r**, t) used earlier, we obtain

$$\overline{\Omega^2}(t) \sim t^{-2D/D_w}, \tag{20}$$

that is, once again by the spectral (fraction) dimension is meant $D_f = 2D/D_w$. In order to change over to the pulsation energy $\overline{u^2}$, we can make use of (2)

$$d\overline{u^2}/dt = -2\nu\overline{\Omega^2}. \tag{21}$$

However, when referring to the fractal regime, this relation must be used in the form

$$d\overline{u^2} = -2\nu\overline{\Omega^2}(dt)_f, \tag{22}$$

and [see (10)]

$$d\overline{u^2} = -2\nu\overline{\Omega^2}t^{-D_r}dt. \tag{23}$$

Finally, using (9) and (20), we obtain

$$\overline{u^2} \sim t^{-D_f/2}. \tag{24}$$

If the universal value $D_f = 4/3$ is used, then from (24) we have

$$\overline{u^2} \sim t^{-2/3}. \tag{25}$$

Recollect that the value $D_f = 4/3$ corresponds to the low-frequency range of the spectrum $S_u \sim \omega^{-5/3}$ (see Sec. 2).

A damping law of the form (25) is indeed found in experiments [4].

We should note that in the MHD experiments, in which a spectrum of the form (2) is observed in the low-frequency range, this spectrum does not necessarily determine the damping of the entire integrated energy of pulsation. However, in the case where the considered change in time is in just this frequency range, the spectrum dynamics is governed by this law. Using the Kolmogorov hypothesis (6) (for $D_f = 4/3$), we obtain (7) from (25). In the previously cited experimental work [10], where observations of a spectrum of the form (2) were described, data on the behavior of S_u with time were also given. These data are shown in Fig. 4. For comparison with (7), the straight line depicting the dependence $S_u \sim t^{-10/9}$ has been drawn in Fig. 4 (x/M ~ t in this figure).

For the degenerate case, with suppressed fractal character (when $D_f \to 2$), we obtain from (24)

$$\overline{u^2} \sim t^{-1}.$$

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KINETICS OF HEAT TRANSFER TO A SPHERICAL PARTICLE FROM A RAREFIED PLASMA.
3. MAXWELLIAN ION APPROXIMATION

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The authors describe the kinetics of heat transfer to a spherical particle from a rarefied plasma with a Maxwellian velocity distribution of molecules, electrons, and ions.

A material particle in a rarefied plasma experiences collisions with molecules, electrons, and ions, resulting in transfer of energy and charge. Plasma electrons recombine on the surface and are absorbed by the particle, and the ions are neutralized by electrons of the material and scattered by the particle surface in the same manner as are incident molecules of the plasma gas. It is important that due to the large difference in the thermal velocities of electrons and plasma ions the particle acquires a negative potential $\varphi_f < 0$ for which the electron and ion charge flux compensate each other, $J_e^-(\varphi_f) = J_i^-(\varphi_f)$. During collisions of electrons and ions with the surface, besides kinetic energy the particle receives energy of the charged states corresponding to the work function Φ_e and the effective ionization energy $I_i - \Phi_e$.

Computations of heat transfer between the particle and the plasma reduce to determining the number flux of plasma particles $J_j^\pm$ of each type and the kinetic energy $E_j^\pm$ transferred by them, from simultaneous solution of the kinetic Boltzmann–Vlasov equation for the velocity distribution function f_j and the Poisson equation for the potential $\varphi(r)$. The main complications in solving the kinetic problem are linked to describing the motion of ions in the attractive field of a charged particle. This arises from the use of simplified distribution models, e.g., the cold ion [1] and the monoenergetic ion [2] approximations, used to describe heat transfer to a particle in [3, 4]. Therefore, it is of interest to analyze beat transfer to a spherical particle from a collisionless plasma at rest ($\ell_j \gg R$) in the more realistic case when the ions, as well as the molecules and electrons, are subject to a Maxwellian velocity distribution in the unperturbed plasma region far from the particle:

$$f_{j\infty} = N_{j\infty}\left(\frac{m_j}{2\pi kT_{j\infty}}\right)^{3/2}\exp\left(-\frac{m_j v^2}{2kT_{j\infty}}\right). \tag{1}$$

For a diffuse law of scattering of molecules and neutralized ions by the particle surface in conditions when thermal-emission processes are not important, the relations for the heat flux $q_j = Q_j/E_j^0$ for each type of plasma particle in dimensionless form are as follows:

$$q_a = 1 - \tau_s, \tag{2}$$

$$q_e = e_e^- + \frac{1}{2} j_e^- w_e, \tag{3}$$

$$q_i = e_i^- + j_i^-\left(\frac{1}{2} w_i - \tau_s\right), \tag{4}$$

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